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The Shock Spectrum: What Is It?

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Mechanical shock loads that are imparted to electrical and mechanical systems can degrade the performance of these systems or even induce failure. It is therefore desirable to develop a methodology to guarantee the reliability of the more critical of these systems when they are subjected to transient mechanical loading. This methodology is provided in Figure 1. A hypothetical example of the application of this methodology follows.

Model/Test Full Scale System	Æ	Predict/Measure Loads to Components	Æ	Certify System via Component Shock Testing

Figure 1: Methodology for Attaining Structural Reliability under Transient Loading

Assume that we want to assure the reliability of a guidance component within a missile system such as shown in Figure 2. We would either model the system analytically, or, preferably, perform a few test flights. We would subsequently model, or, again, preferably measure, the transient acceleration loads imparted to the guidance component at its mounting support. Assuming that the component functioned properly, we could set up a program to perform additional flight tests routinely in such a way as to maintain an ongoing assessment of the component's reliability.

Figure 2: Theater Defense Missile (Lockheed Martin Missiles and Fire Control)

However, such testing would quickly become too expensive. Therefore, to instill ongoing confidence that the component will function properly after encountering mechanical shock loading in its use environment, the design engineer needs a laboratory test method.

A problem exists in the preceding example since the transient acceleration-time histories that were measured at the component mount are unique to the missile system launch-and-flight environment. These time-histories typically cannot be replicated using laboratory shock equipment. The challenge is then to create in the test laboratory a qualification shock environment for the guidance component that exceeds the component's operational environment in some sense. The shock response spectrum (SRS), first presented by Biot[1] in 1933, is a widely accepted tool used to develop these laboratory tests.

We assume that the component of interest can be modeled by a continuous series of single-degree-of-freedom (SDOF), second-order systems (oscillators), each increasing in natural frequency ($f_n = [1/(2\pi)]\sqrt{k/m}$). Figure 3 represents this independent set of oscillators. Their increase in natural frequency is pictorially represented by a decreasing mass size from left to right. By convention, we assign to each oscillator a ratio of critical viscous damping of $\xi = 0.03$. Figure 4 provides enhanced definition of one of these oscillators.

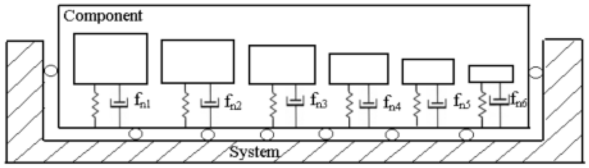

Figure 3: Component Represented by SDOF Oscillator Array

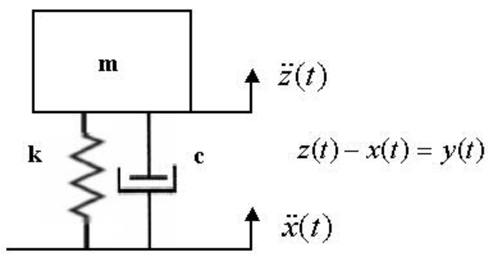

Figure 4: Individual Oscillator with Coordinates Defined

Focusing on the missile in Figure 2, we will conjecture that at release from its launch canister we measure an acceleration input to a guidance component as in Figure 5. We will then analytically input this acceleration $[\ddot{x}(t)]$ into the base of one of the oscillators (Assume #1). We will then calculate the maximum absolute acceleration time history response (Figure 6a) of oscillator #1 to $\ddot{x}(t)$ and eventually plot this point at frequency f_{n1}. This plotted response point will be $\ddot{z}(t) = \ddot{y}(t) + \ddot{x}(t)$. We will then advance to oscillator #2, perform the same calculation (Figure 6b), and plot the result at frequency f_{n2}. As the oscillators become more and more stiff ($f_n = \infty$), they will eventually follow $[x(\ddot{t})]$ exactly, and the maximum response (Figure 6c) will be the same as the maximum peak value in Figure 5.

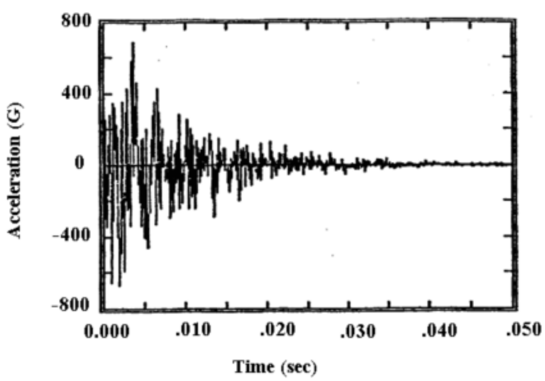

Figure 5: Component Input

Figure 7 is then the envelope resulting from plotting the individual responses of an infinite set of oscillators to the acceleration input of Figure 5. This plot of the maximum absolute value of each acceleration response $\ddot{z}(t)$ for each oscillator as a function of frequency f_n represents the most common two-dimensional shock spectrum. In some instances, where a more appropriate component failure model holds, the maximum absolute velocity or displacement spectrum might be plotted as an alternative. As long as the guidance component can be modeled as Figure 3, and its damage can be related to its peak response experienced as a result of the shock, the shock spectrum provides an indication of the damage potential of the shock being analyzed.

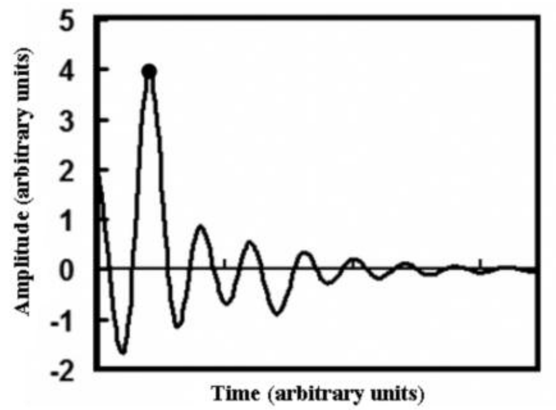

Figure 6a: Oscillator #1 response at f_{n1} = f(1)

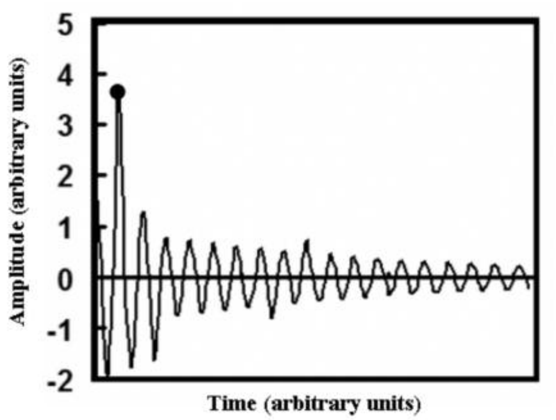

Figure 6b: Oscillator #2 response at f_{n2} = f(2)

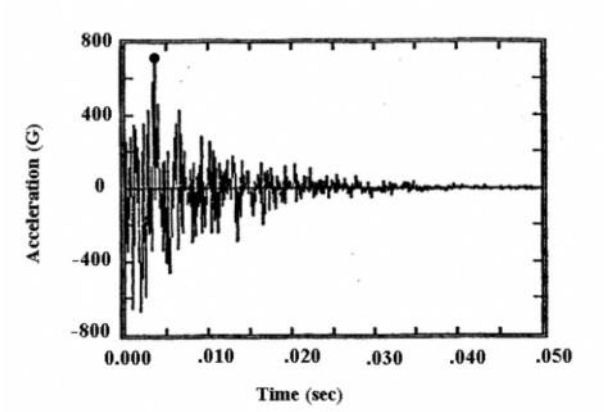

Figure 6c: Oscillator response at $f_{n\infty} = f(\infty)$

Figure 6: Example of Response of Individual Oscillators in Figure 3 to Component Input in Figure 5

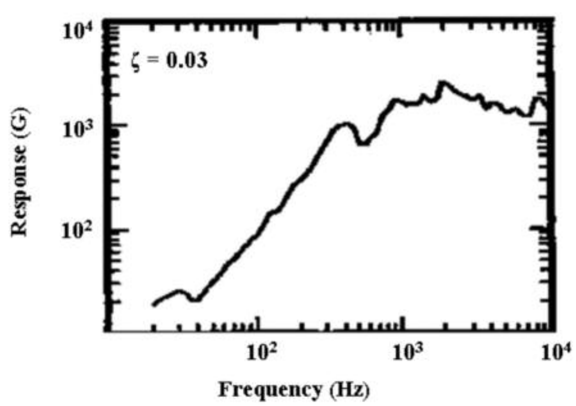

Figure 7: Envelope of Maximum Response vs. Frequency of an Infinite Number of Oscillators to the Shock in Figure 5

The next challenge is to establish an equivalent qualification-shock for the test laboratory. The commonly made assumption is that any qualification shock is acceptable as long as its shock spectrum fully encompasses the measured field environment. Haversine shapes are classical pulses that are called out in shock testing and that can be approximated on laboratory shock machines. A haversine pulse of amplitude A and period T would analytically be expressed as:

$$\ddot{x}(t) = (A/2)[1 - \cos(2\pi t/T)], \quad 0 \le t \le T \qquad (1)$$

We want to select a haversine pulse whose shock spectrum will encompass but not greatly exceed that of the field shock (per Figure 7). Figure 8a shows such a pulse and Figure 8b shows how its shock spectrum envelops the shock spectrum of the field pulse. Thus, for our example, a 2000G-amplitude haversine pulse of 0.5 milliseconds (0.0005 seconds) duration will become the qualification shock for our guidance component. Since Figure 5 has both positive and negative halves, the haversine pulse needs to be applied in the test laboratory in both directions.

Much more could be written about the shock spectrum technique and variations associated with it. The technique satisfies the requirement to provide a simple means to characterize a shock environment, and, since it is an enveloping technique, it also enables different shock events to be combined into a single environment. However, it remains controversial because acceleration pulses that differ greatly in amplitude, frequency content, and duration can produce equivalent shock spectra. For example, the previous haversine pulse can be seen to have a finite velocity change associated with it while the field pulse does not.

Nevertheless, in spite of any controversy associated with the shock spectrum method, it remains firmly entrenched as the principal tool for laboratory shock testing, and, as such, the method is a key element in the process of assuring system reliability in field applications.

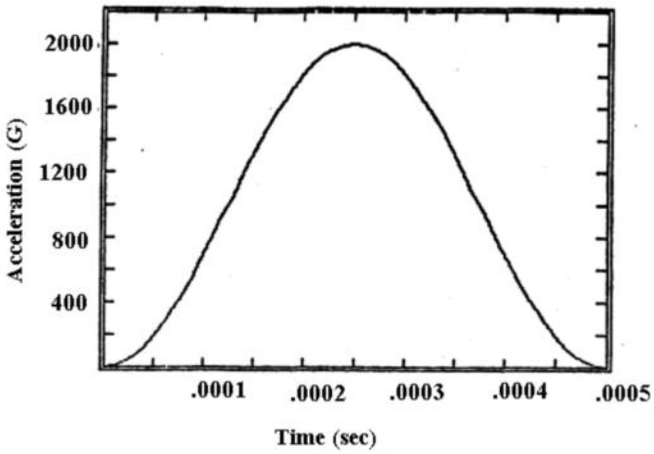

Figure 8(a)

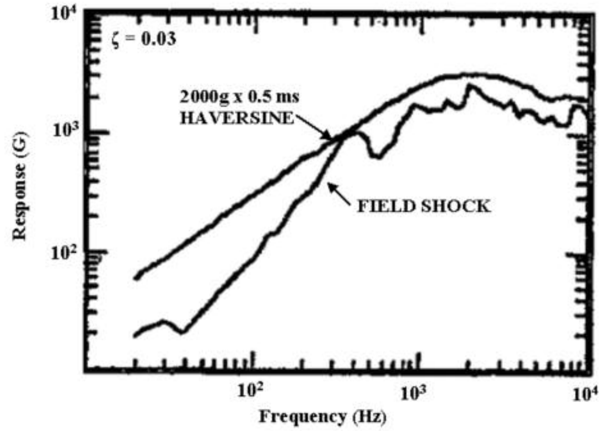

Figure 8(b)

Figure 8: Haversine Pulse and Enveloped Shock Spectrum of Field Shock

REFERENCE

1. Biot, M. A., "Theory of Elastic Systems Vibrating Under Transient Impulse With an Application to Earthquake Proof Buildings", Proceedings of the National Academy of Sciences, Vol. 19, No. 2, 1933, pp. 262-268.

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